R. H. MANN.

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The Association
of
Engineering and Shipbuilding Draughtsmen.

DESIGN OF SHORT OVERHEAD TRANSMISSION LINES.

By J. W. PEACOCK

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INTRODUCTION.

It is hoped that this pamphlet will provide a solid grounding in the principles of overhead transmission line design.

A knowledge of simple A.C. vector theory is assumed in the section on line performance, but otherwise the mathematics have been kept as simple as possible.

The subject matter has been limited to "short" lines and wooden pole construction and this class of line has been treated fairly thoroughly.

All examples given are part of the same design and particulars have been collected together at the end of the book for ease of consultation.

Once again, it is the author's sincere hope that this pamphlet will be of practical use, and encourage the reader to further study.

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ESTIMATING CONDUCTOR SIZE.

Unless precise load conditions are known, as in the case of interconnectors or extensions to existing loads, any calculations made regarding the conductor size must be based on assumptions.

Economic conditions also affect the conductor size. If there is an excess of generating capacity over demand, a large energy loss is of little account and a line which gives a minimum capital cost would be chosen. If, however, the excess generating capacity is small or non-existent, all energy losses are equivalent to units sold, in which case a larger conductor would be chosen to give a smaller energy loss even though the capital cost is increased.

The two methods described below are found in practice to give good results, and in many cases the minimum conductor size can be found by considering the maximum permissible volt drop.

Mean Annual Current.

This method involves finding the steady current which will produce the same annual energy loss as the actual load current. If a typical daily load cycle is obtainable considerable accuracy may be obtained.

The method of working is as follows :—

The daily load curve is taken and blocked off as shown in Fig. 1, into periods of I_1 per cent. of full load current for T_1 hours, I_2 per cent. of full load current for T_2 hours, etc., until the whole curve is taken into account.

$$\text{Heating factor } F = \sqrt{\frac{I_1^2T_1 + I_2^2T_2 + I_3^2T_3 + \ldots\ldots}{T_1 + T_2 + T_3 + \ldots\ldots}}$$

and the mean annual current = 100% full load current × F.

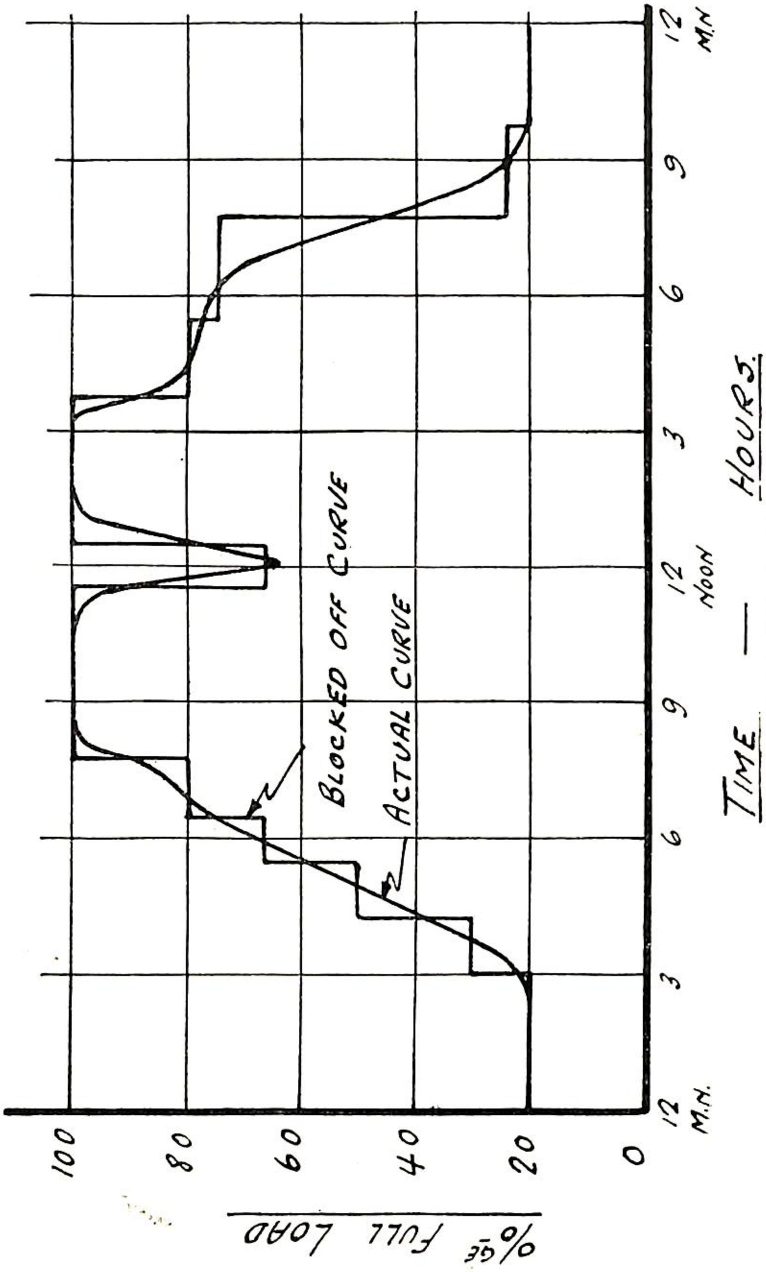

Fig. 1—Typical Daily Load Curve,

For the load curve shown in Fig. 1 the working is as shown :—

Load %	Load^2	Hours	Load^2 × Hours
100	1·00	7·0	7·0
80	·64	3·0	1·92
74	·55	2·25	1·24
66	·44	2·0	·88
50	·25	1·25	·313
30	·09	1·25	·133
24	·058	2·0	·116
20	·04	5·25	·210
		24·0	11·792

$$\text{Heating factor} = \sqrt{\frac{11{\cdot}792}{24}}$$

$$= {\cdot}70$$

The mean current is therefore 70% of the peak current.

Equivalent Hours.

This method involves finding the number of hours per annum at peak current that will give the same energy loss as the actual load cycle.

This is done in a similar manner to the mean annual current method by taking percentage loads over various periods and finding the load factor which is defined as the average current over a certain period divided by the full load current :—

$$\text{Load factor} = \frac{I_1T_1 + I_2T_2 + I_3T_3 + \ldots\ldots}{T_1 + T_2 + T_3 + \ldots\ldots} \times \frac{1}{I}$$

where I_1, I_2, etc., are the percentage full load currents for times T_1, T_2, etc., and I is the full load current.

By analysing Fig. 1 in this manner the load factor is found to be ·552.

An equation which the author has used successfully in many designs, for finding the equivalent hours from the load factor is :—

$$E = (0{\cdot}4 \text{ to } 0{\cdot}7) \times (\text{load factor})^2 + (0{\cdot}6 \text{ to } 0{\cdot}3) \times (\text{load factor}).$$

The coefficient depends on the shape of the load curves, *e.g.*, for a peaked curve the coefficient would be 0·7 and 0·3 respectively and for a fairly flat curve they would be 0·4 and 0·6 respectively ; the coefficients always adding up to 1·0.

E is a multiplying factor, and the equivalent hours are 8760 × E.

Returning to our previous example, the coefficient may be taken as 0·5 as we have a fairly flat load curve.

$$\begin{aligned} \text{Then } E &= 0{\cdot}5 \times ({\cdot}552)^2 + 0{\cdot}5 \times {\cdot}552 \\ &= 0{\cdot}428 \end{aligned}$$

$$\text{The equivalent hours} = 0{\cdot}428 \times 8760$$
$$= 3750 \text{ hours.}$$

The line could operate therefore for 3750 hours at full load to produce the same energy loss as the actual load cycle.

Example.

Estimate the conductor size for a transmission line, 20 miles long to carry 2500-kW. at 0·8 p.f. 33,000 volts. The conductor spacing is 6 ft. and the load cycle is as shown in Fig. 1.

$$\text{Full load current} = \frac{2500 \times 1000}{\sqrt{3} \times 33{,}000 \times 0{\cdot}8} = 54{\cdot}7 \text{ amps.}$$

Allow say 4% voltage drop at this load to allow for future extensions,

$$\text{Volt drop} = \frac{33{,}000}{\sqrt{3}} \times \frac{4}{100} = 763 \text{ volts.}$$

$$\text{Line impedance} = \frac{763}{54{\cdot}7 \times 20 \times 1{\cdot}76} = 0{\cdot}397 \text{ ohms/1000 yds.}$$

A 7/·152 copper conductor has an impedance of 0·402 ohms/1000 yards and will be suitable from the point of view of voltage drop. This conductor must now be justified economically and this can be done by using the mean annual current for calculating losses.

Full load current = 54·7 amps.

The heating factor has been found above and is 0·70 so that :—

$$\text{The mean current} = 54{\cdot}7 \times /{\cdot}70$$
$$= 38{\cdot}3 \text{ amps.}$$

The following calculations will be easier to analyse if set out in tabular form as shown below.

Size of conductor, ...	3/·180	7/·136	7/·152	7/·166
Resistance of 20 miles of one conductor, ...	11·718	8·786	7·033	5·899
I²R losses of 3 conductors kW.h/annum, ...	463,599	347,600	278,246	233,382
Cost of I²R losses/annum at 0·5d per unit, ...	£964	£723	£579	£485
Weight of 20 miles of 3 conductors in tons,	43·29	57·51	71·82	85·65
Cost of conductor at £200 per ton, ...	£8658	£11,402	£14,364	£17,130
Fixed charge at 10% cost, ...	£866	£1140	£1436	£1713
Total cost per annum = cost of losses plus fixed charge, ...	£1830	£1863	£2015	£2198

Under present day conditions when every unit can be sold, the line giving lowest I^2R losses consistant with a reasonable price is chosen. In this case 7/·136 or 7/·152 conductor would be chosen as the total annual cost does not vary more than about £200 from the 3/·180 conductor and the annual saving on losses by using a 7/·152 conductor rather than a 3/·180 conductor is £380. A 7/·152 conductor will also permit further load increases to be carried safely.

I^2R losses are calculated from the mean annual current multiplied by the resistance of **three** conductors multiplied by 8760, *i.e.*, the number of hours per annum.

e.g., for 3/·180 conductor

$$I^2R \text{ losses} = \frac{(38\cdot3)^2 \times (11\cdot718 \times 3) \times 8760}{1000}$$

$$= 463{,}599 \text{ kW.h/annum.}$$

ELECTRICAL DESIGN AND PERFORMANCE.

For overhead lines less than about 50 miles in length the effect of capacitance may be neglected, as the increase in voltage drop for a 50 mile line is only 0·5% if capacitance is included.

A line can then be represented by a simple series circuit composed of resistance and inductance as shown in Fig. 2.

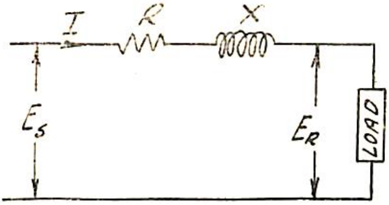

Fig. 2—Transmission Line Equivalent Circuit.

Inductance is calculated from $L = k + 0\cdot741 \log_{10} D/r$ millihenries per mile.

where k is a constant :—

3 strand conductors $k = 0\cdot125$
7 ,, ,, $k = 0\cdot103$
19 ,, ,, $k = 0\cdot089$
37 ,, ,, $k = 0\cdot085$
61 ,, ,, $k = 0\cdot083$
Solid $k = 0\cdot080$
r is the conductor diameter

and D is the equivalent conductor spacing and is equal to

$$D = \sqrt[3]{A \times B \times C}$$

where A, B and C are the individual conductor spacings.

The circuit impedance comprises all generator, transformer and line impedances and for calculation purposes it is more convenient to have all values phase to neutral.

The vector diagram for a short line is shown in Fig. 3.

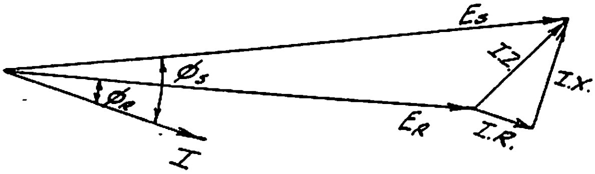

Fig. 3—Transmission Line Vector Diagram.

Normal operation practice is to keep the receiving end voltage as steady as possible by varying the sending end voltage in accordance with the load.

The sending end voltage is approximately

$$E_S = E_R + (I.R. \cos \phi_R + I.X. \sin \phi_R)$$

and in practice this gives results quite accurately enough. The sending end power factor can be found from

$$\tan \phi_S = \frac{\sin \phi_R + IX/E_R}{\cos \phi_R + IR/E_R}$$

Regulation is defined as the percentage rise in voltage at the receiving end when load is thrown off, the sending end voltage being constant.

$$\%^{ge} \text{ Regulation} = \frac{E_S - E_R}{E_R} \times 100$$

$$= \frac{I.R. \cos \phi_R + I.X. \sin \phi_R}{E_R} \times 100$$

In practice, loads and power factors are continuously varying and as it is inconvenient to calculate the sending end voltage, etc., each time a voltage change is made, a chart should be made up showing load, sending end voltage, and power factor for any load condition.

Essentially the regulation chart, as it is called, is merely an extension of the simple vector diagram. I.R., I.X. and I.Z. are directly proportional to the load and rotate according to the power factor of the load. The method of constructing a regulation chart is best shown by means of an example.

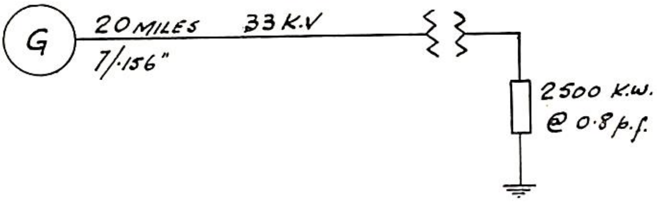

Fig. 4—Transmission Line.

Example.

Construct a regulation chart for the line shown in Fig. 4.

Conductor resistance = 0·1998 ohms, per 1000 yds.
Conductor reactance = 0·3488 ohms. per 1000 yds.
Transformer resistance = 9·5 ohms. referred to H.T. side.
Transformer reactance = 16 ohms. „ „ „ „
Total resistance = $(0{\cdot}1998 \times 1{\cdot}76 \times 20) + 9{\cdot}5 = 16{\cdot}53$ ohms.
Total reactance = $(0{\cdot}3488 \times 1{\cdot}76 \times 20) + 16 = 28{\cdot}28$ ohms.

$$\text{Full load current} = \frac{2500 \times 1000}{\sqrt{3} \times 33{,}000 \times 0{\cdot}8} = 54{\cdot}7 \text{ amps.}$$

I.R. = $54{\cdot}7 \times 16{\cdot}53$ = 903·2 volts.
I.X. = $54{\cdot}7 \times 28{\cdot}28$ 1547·6 volts.

Assuming that the receiving end voltage is to be kept constant at 33-kV. or $33/\sqrt{3} = 19{\cdot}05$ kV. phase to neutral, E_R can be drawn out to scale to represent 19·05 kV.

The voltage drop vectors represented by I.R. and I.X. may now be drawn in to scale for unity power factor, *i.e.*, I.R. is in the same straight line as E_R as shown in Fig. 5. I.X. is drawn in at 90° to I.R. and the total impedance vector is drawn to close the volt drop triangle.

It is clear that these voltage drop vectors are directly proportional to the current and consequently to the load and thus to a different scale can represent power.

The full load at 0·8 p.f. is 2500 kW. and at unity power factor this represents $\frac{2500}{0{\cdot}8} = 3125$ kW., thus the I.Z. vector can be scaled to represent 3125-kW. instead of a voltage drop.

A squared chart can now be made up to this kW. scale, as shown, to give kW. and lagging and leading K.V.A.R.

As the power factor varies the current will lag or lead the voltage and the impedance triangle will change position by the corresponding out of phase angle. To show this on the chart the angles corresponding to particular power factors are calculated and power factor lines

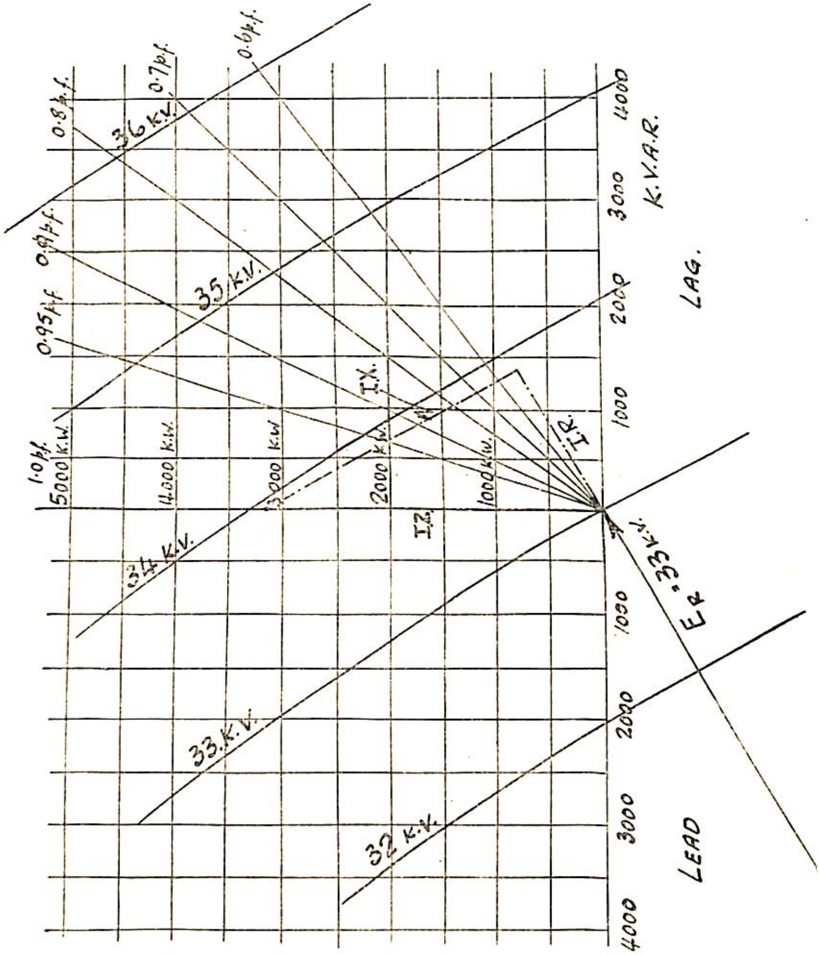

Fig. 5—Regulation Chart.

at the corresponding angle from the I.Z. or kW. scale are drawn. This is shown in Fig. 5 for various lagging power factors only.

To complete the chart, voltages must be shown, and these are arcs of circles scaled to the phase to neutral values as shown. These voltage lines are actually scaled as phase to neutral values but are proportional to the between phase voltage and are so marked on the chart.

An example will show the use of the chart.

Example.

What is the sending end voltage and percentage regulation for the above line at 2500 kW. 0·8 p.f., the receiving end voltage being constant at 33 kV.? What is the transmission efficiency?

The 2500 kW. mark on the scale is projected across to the 0·8 p.f. line, and the voltage is scaled off between the 34 kV. and 35 kV. lines.

E_S = 34·635 kV.
kW. = 2500
K.V.A.R. = 1870 lagging.

$$\%^{ge}\text{ Regn.} = \frac{34{\cdot}635 - 33}{33} \times 100 = 4{\cdot}95\%.$$

These results can be checked by calculation.

$$\begin{aligned} E_S &= E_R + (\text{I.R.}\cos\phi_R + \text{I.X.}\sin\phi_R) \\ &= 33{,}000 + (903 \times 0{\cdot}8 + 1547 \times 0{\cdot}6) \\ &= 33{,}000 + 1650 \\ &= 34{\cdot}65\text{ kV.} \end{aligned}$$

$$\%^{ge}\text{ Regn.} = \frac{34{\cdot}65 - 33}{33} \times 100 = 5\%$$

The K.V.A.R. may be calculated by finding the K.V.A., *i.e.*, $\frac{\text{kW.}}{\text{Power factor}}$ and applying Pythagoras's theorem to the right angled triangle obtained

$$\text{K.V.A.R.} = \sqrt{\left(\frac{2500}{0{\cdot}8}\right)^2 - 2500^2} = 1875\text{ lagging.}$$

The results from the chart are very close to the calculated results and, of course, depend on the accuracy with which the chart is drawn.

The transmission efficiency may be found by calculating the I^2.R. loss per phase.

$$\text{Received power per phase} = \frac{2500}{3} = 833{\cdot}3 \text{ kW.}$$

$$I^2R \text{ loss per phase} = \frac{54{\cdot}7^2 \times 16{\cdot}53}{1000} = 49{\cdot}5 \text{ kW.}$$

$$\text{Efficiency} = \frac{833{\cdot}3}{833{\cdot}3 + 49{\cdot}5} = 94{\cdot}4\%$$

Example.

What power can be delivered at 0·8 p.f. if E_s is 35 kV. and E_R is 33 kV.

The point of intersection of the 0·8 p.f. line and the 35 kV. line is projected to the kW. and K.V.A.R. scales.

kW. = 3060
K.V.A.R. = 2300 lagging.

SAG AND TENSION CALCULATIONS.

The two principal methods of calculating sags in overhead conductors are the parabola and catenary methods, neither of which are absolutely correct. However, for spans of less than 1000 feet and where the sag is small compared with the span the parabola method gives sufficiently accurate results. For long spans with conductors consisting of many strands, the conductor approximates to the perfectly flexible condition required for the catenary solution.

The parabola method has the advantage of being very much simpler to handle than the catenary method ; the mathematical operations being of the simplest kind.

The equations to the parabola which will be required may be found in any text-book on mathematics and will be summarised below for convenience.

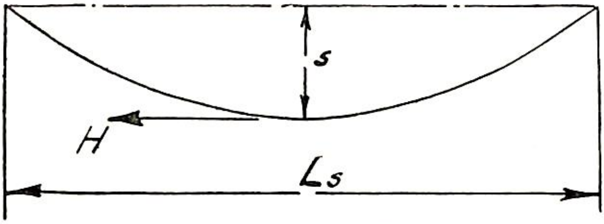

Fig. 6—Parabola.

Sag at centre span $= s = \dfrac{w \times L_s^2}{8 \times H}$

Length of conductor in span $= L = L_s + \dfrac{8 \times s^2}{3 \times L_s}$

Unstressed length of conductor in span $= L_u = \dfrac{L}{1 + H/E \times a}$

Elongation of conductor due to tension $= e = \dfrac{H \times L_u}{E \times a}$

Where :—

w = Weight of conductor per foot run in lbs.
E = Modulus of elasticity of conductor = 18×10^6 lbs. per sq. in.
a = Cross sectional area of conductor in sq. ins.

and H in lbs. and L_s and s in feet are as shown in Fig. 6.

Whilst erecting a transmission line, various conditions of temperature and wind obtain from day to day. To enable the erector to proceed with the work with as little trouble as possible, some means must be provided to give the erector the correct sag and conductor tension for the conditions prevailing. This information is usually given on charts as shown in Figs. 8 and 9.

The range of loading conditions as specified by the Electricity Commissioners is very wide, being :—

Worst Load Conditions.

The conductor is assumed to be covered with ice of $\frac{3}{8}''$ radial thickness for H.T. lines, and 8 lbs. per sq. ft. wind pressure on projected area of conductor and ice at a temperature of 22°F.

Maximum Sag Conditions.

The sag, when calculated at 122°F. in still air must exceed the sag under the worst loading conditions. This is the sag used for computing ground clearances and support heights.

The erection chart must cover all these extremes of ice, wind, and temperature for a full solution and the method of producing such a chart will be shown below by means of an example.

Example.

Construct erection charts for the following line :—

Ruling span—350 feet. Conductor size—7/·152″.

Overall diameter of conductor	= 0·456 ins.
Area of conductor	= 0·12489 sq. ins.
Breaking load	= 7232 lbs.
Weight	= 0·508 lbs. per foot run.
Minimum safety factor	= 2

Conductor Loads.

The wind load per foot run may be found by the following equation :

$$\text{Wind load} = \frac{P\ (D + 2t)}{12}$$

where P = wind pressure in lbs. sq. ft.
D = conductor diameter.
t = radial thickness of ice.

and for a wind pressure of 8 lbs. per sq. ft. this becomes

$$\text{Wind load} = \tfrac{2}{3}\ (D + 2t) \text{ lbs. per foot run.}$$

$$\text{Wind load} = \tfrac{2}{3}\ (\cdot456 + \cdot75)$$

$$= 0{\cdot}804 \text{ lbs. per foot run.}$$

The ice load can be calculated from :—

$$\text{Ice load} = \frac{W\pi t\ (D + t)}{144} \text{ where } W = \text{weight of ice per cu. ft.}$$

and allowing 57 lbs. per cu. ft. this becomes :—

$$\text{Ice load} = 0{\cdot}4665\ (D + t).$$

$$\text{Ice load} = 0{\cdot}4665\ (\cdot456 + \cdot375).$$

$$= 0{\cdot}388 \text{ lbs. per foot run.}$$

These loads are assumed to act at 90° to each other, the ice load acting in the same direction as the conductor load. (See Fig. 7). The total load will then be :—

$$\text{Conductor load} = \sqrt{0{\cdot}804^2 + (\cdot388 + \cdot508)^2}$$

$$= 1{\cdot}204 \text{ lbs. per foot run.}$$

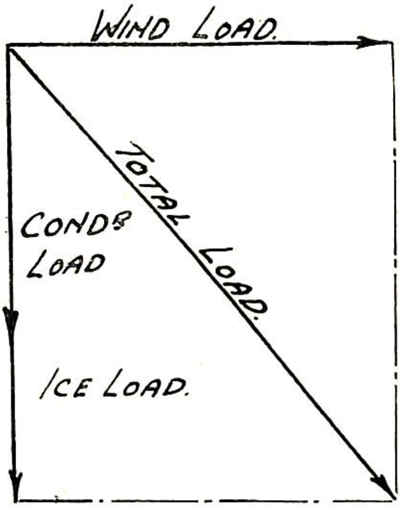

Fig. 7—Conductor Loading.

Worst Load Conditions.

Taking the minimum safety factor of 2, the maximum working tension will be 3616 lbs., so that a tension of 3500 lbs. will be chosen for preliminary work. With this tension the sag will be :—

$$\text{Sag} = \frac{w\,L_s^2}{8\,H} = \frac{1{\cdot}204 \times 350 \times 350}{8 \times 3500} = 5{\cdot}268 \text{ ft.}$$

The length of conductor in the span $= L = L_s + \dfrac{8 \times s^2}{3 \times L_s}$

$$= 350 + \frac{8 \times 5{\cdot}268 \times 5{\cdot}268}{3 \times 350}$$

$$= 350{\cdot}2114 \text{ ft.}$$

The unstressed length of the conductor assuming all tension is removed $= \dfrac{L}{1 + \dfrac{H}{E \times a}}$

$$= \frac{350{\cdot}2114}{1 + \dfrac{3500}{18 \times 10^6 \times {\cdot}12489}}$$

$$= 349{\cdot}6659 \text{ ft.}$$

This is the unstressed length of the conductor at 22°F., the temperature specified for the worst loading conditions, and the unstressed lengths at other temperatures must now be found. The temperature coefficient of linear expansion of copper is 0·0000094 ft. per °F.

Temp. °F.	Unstressed Length in Span.
22	$349{\cdot}6659\ (1 + {\cdot}0000094 \times 0) = 349{\cdot}6659$ ft.
32	$349{\cdot}6659\ (1 + {\cdot}0000094 \times 10) = 349{\cdot}6988$ ft.
60	$349{\cdot}6659\ (1 + {\cdot}0000094 \times 38) = 349{\cdot}7907$ ft.
90	$349{\cdot}6659\ (1 + {\cdot}0000094 \times 68) = 349{\cdot}8893$ ft.
122	$349{\cdot}6659\ (1 + {\cdot}0000094 \times 100) = 349{\cdot}9946$ ft.

The effect of tension must now be found and this is done by taking the unstressed length at a particular temperature and adding the extension caused by varying tensions.

22°F. Temperature.

The elongation of the conductor per 1000 lbs.

$$= \frac{H \times L_u}{E \times a}$$

$$= \frac{1000 \times 349{\cdot}6659}{18 \times 10^6 \times {\cdot}12489}$$

$$= 0{\cdot}1552 \text{ ft.}$$

Tension.	Length of Conductor in Span.
4000	349·6659 + ·6208 = 350·2867 ft.
3500	349·6659 + ·5432 = 350·2091 ft.
3000	349·6659 + ·4656 = 350·1315 ft.
2500	349·6659 + ·3880 = 350·0539 ft.

Now it can be shown that the sag

$$s = 0{\cdot}5 \sqrt{1{\cdot}5\ L_s\ (L-L_s)}$$

and for a 350 ft. span this becomes $s = 0{\cdot}5 \sqrt{525\ (L-350)}$ where L is the length of conductor in the span given in the table above.

Tension.	Sag.
4000	$\cdot 5 \sqrt{525 \times \cdot 2867} = 6{\cdot}235$ ft.
3500	$\cdot 5 \sqrt{525 \times \cdot 2091} = 5{\cdot}240$ ft.
3000	$\cdot 5 \sqrt{525 \times \cdot 1315} = 4{\cdot}159$ ft.
2500	$\cdot 5 \sqrt{525 \times \cdot 0539} = 2{\cdot}659$ ft.

The sags for temperatures of 32°, 60°, 90° and 122°F. must now be calculated in exactly the same manner but using the appropriate unstressed length for each set of calculations. The working will not be shown here as it is the same process as shown above for 22°F., but the complete solution is shown in graphic form in Fig. 8.

To complete the solution the sags under particular load conditions are required to be found.

Ice and Wind Load.

The combined ice and wind load gives a total conductor loading of 1·204 lbs. per foot run and the sag is :—

$$s = \frac{1{\cdot}204 \times 350 \times 350}{8 \times H} = \frac{18{,}436}{H} \text{ ft.}$$

Tension.	Sag.
3250	18,436/3250 = 5·673 ft.
3500	18,436/3500 = 5·267 ft.
3750	18,436/3750 = 4·916 ft.

Ice Load Only.

The total conductor loading in this case is 0·896 lbs. per foot run and the sag is :—

$$s = \frac{{\cdot}896 \times 350 \times 350}{8 \times H} = \frac{13{,}720}{H} \text{ ft.}$$

Tension.	Sag.
2750	13,720/2750 = 4·989 ft.
3000	13,720/3000 = 4·573 ft.
3250	13,720/3250 = 4·221 ft.

No Ice or Wind Load.

The total load in this case is due to the conductor weight only and is 0·508 lbs. per foot run and the sag is :—

$$s = \frac{{\cdot}508 \times 350 \times 350}{8 \times H} = \frac{7778{\cdot}8}{H} \text{ ft.}$$

The points taken for plotting this curve are taken closer together, as this curve is used for erection in still air and a fair degree of accuracy is required.

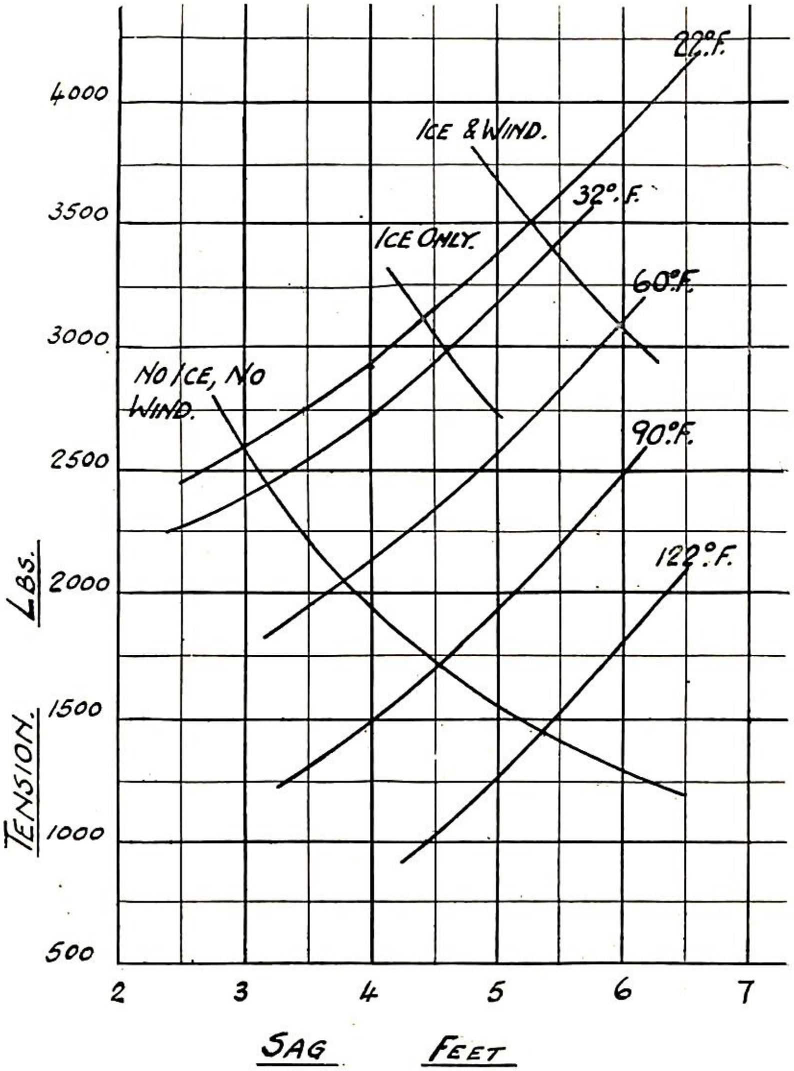

Fig. 8—Sag—Tension Curves.

Tension.	Sag.
1200	7778·8/1200 = 6·482 ft.
1400	7778·8/1400 = 5·556 ft.
1600	7778·8/1600 = 4·862 ft.
1800	7778·8/1800 = 4·321 ft.
2000	7778·8/2000 = 3·890 ft.
2200	7778·8/2200 = 3·536 ft.
2400	7778·8/2400 = 3·241 ft.
2600	7778·8/2600 = 2·991 ft.
2800	7778·8/2800 = 2·777 ft.

It is clear that the only points of any practical value are the points of intersection of the various sets of curves, and using these values a temperature-tension curve may be drawn, taking all values from the no ice, no wind curve in Fig. 8.

Tension.	Temp. °F.
2600	22
2440	32
2050	60
1710	90
1450	122

With the parabola solution the sag is proportional to the square of the span length, and consequently the sags for longer or shorter spans may be found by simple ratio, *i.e.*,

$$s_2 = s_1 \times \frac{L_2^2}{L_1^2}$$

and curves may be drawn up showing the sag at various spans and temperatures. For erection purposes these curves are usually shown on the same paper as the temperature—tension curves. This ratio method may be safely used up to twice the ruling span length.

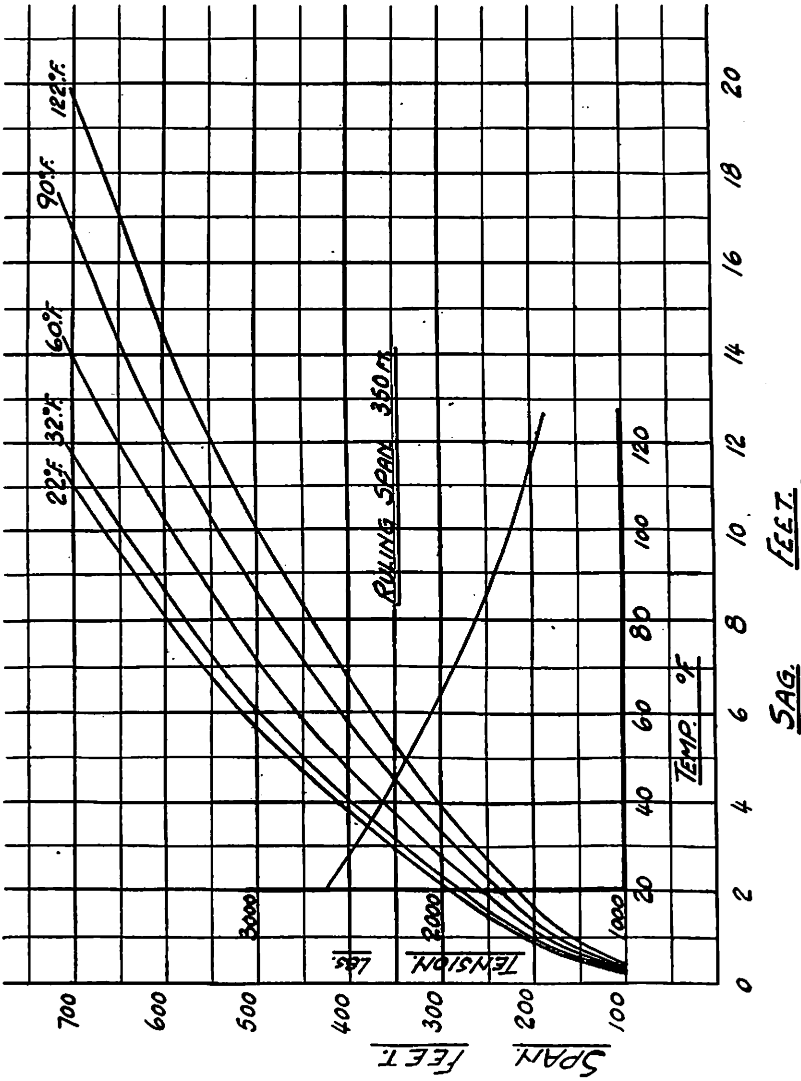

Fig. 9—Erection Curves.

Span Ft.	Sag-Feet.				
	22°F.	32°F.	60°F.	90°F.	122°F.
100	·227	·242	·287	·344	·407
200	·903	·96	1·14	1·37	1·62
350*	2·99	3·18	3·78	4·53	5·35
500	5·64	6·00	7·13	8·55	10·09
600	8·13	8·64	10·27	12·31	14·54
700	11·06	11·76	13·98	16·75	19·79

* Ruling Span, 350 ft.

Lines on Grades.

Fig. 10 shows a diagram of a line erected on a grade. The mathematical treatment is the same as for a larger theoretical span with supports at the same level, the span of which is given by :—

$$L_2 = L_s + \frac{2 H h}{W L_s}$$

where L_s is the horizontal distance between supports and equals $l \cos \theta$ and H is the horizontal conductor tension lbs.

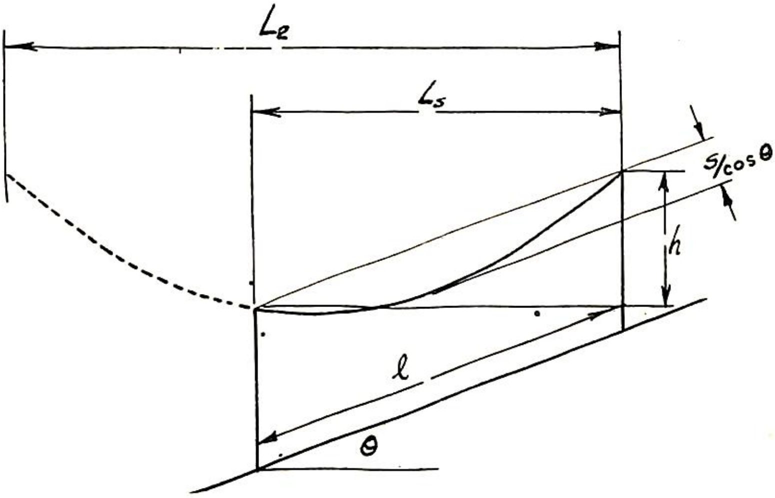

Fig. 10—Lines on Grades.

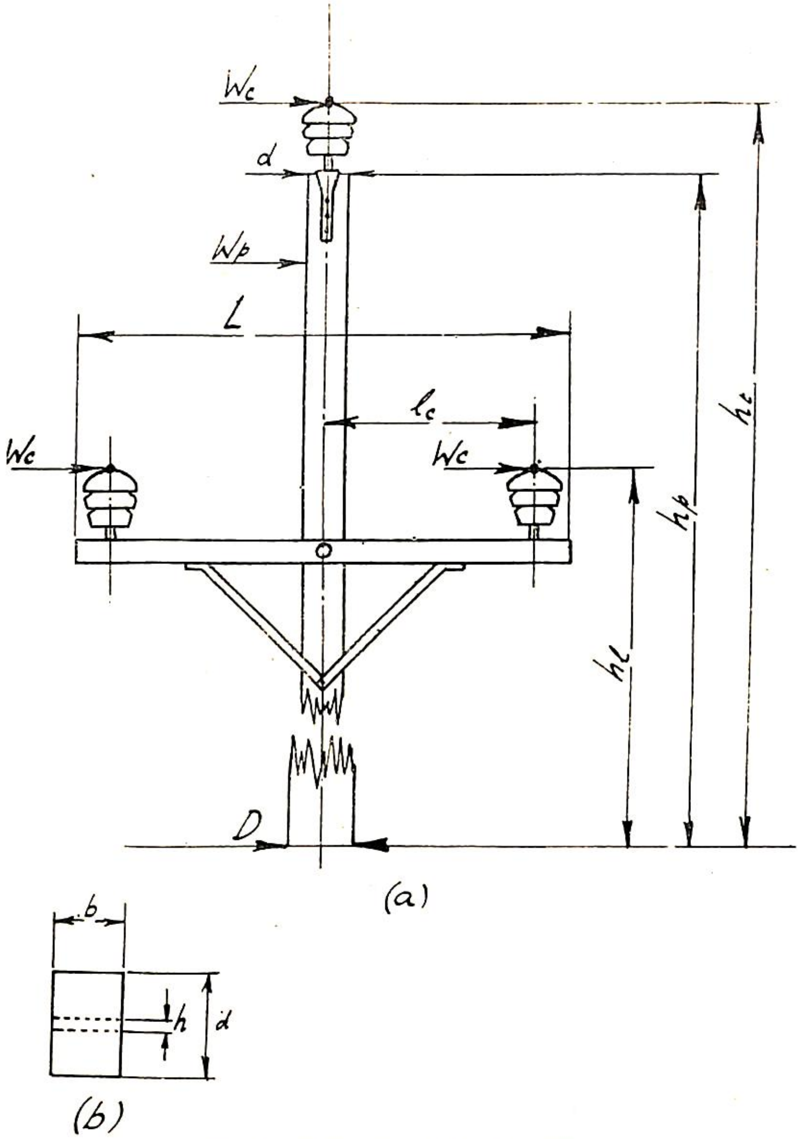

Fig. 11—Typical Wood Pole Construction.

SUPPORT DESIGN.

Pole Design.

Wood poles are usually made of red fir impregnated with creosote and having an ultimate fibre stress of about 7800 lbs. per sq. in. A typical wood pole construction is shown in Fig. 11. The conductors are carried on pin insulators, one of which is on the pole top and the other two on a crossarm.

The load on a pole is transverse only and is due to the wind pressure on the pole and conductors. There is no longitudinal load as there should not be any difference in conductor tension in adjacent spans. Worst loading conditions are assumed for calculating loads, *i.e.*, 8 lbs. per sq. ft. wind pressure on the pole and ice covered conductors, and $\frac{3}{8}''$ radial thickness of ice on the conductors. The minimum safety factor as specified by the Electricity Commissioners is 3·5 for poles and crossarms.

The length of the pole and height of conductor supports are fixed by the minimum ground clearance for the conductor at mid span and the sag at 122°F. in still air.

The total bending moment at ground level is the sum of the bending moments due to wind pressure on the pole, and wind pressure on the conductors, and these may be found :—

Bending moment at ground level due to wind load on top conductor $= M_t = W_c \times h_t \times L$ ft. lbs.

Bending moment at ground level due to wind load on two lower conductors $= M_l = W_c \times h_l \times L \times 2$ ft. lbs.

Bending moment at ground level due to wind load on pole

$$= M_p = \frac{W_p \times h_p^2 \times (D + 2d)}{72} \text{ ft. lbs.}$$

Where W_p = wind pressure on pole in lbs. per sq. in.
W_c = wind pressure on conductor in lbs. per ft. run.

and D, d, h_l, h_p, h_t, are as shown in Fig. 11.

L is the equivalent span in feet and is equal to half the sum of the spans on either side of the pole under consideration. (See Fig. 12).

The section modulus at ground level for a circular pole is

$$Z = \frac{\pi D^3}{32}$$

and the fibre stress is

$$F = \frac{M \times 12}{Z} \text{ lbs. per sq. in.}$$

where $M = M_t + M_l + M_p$

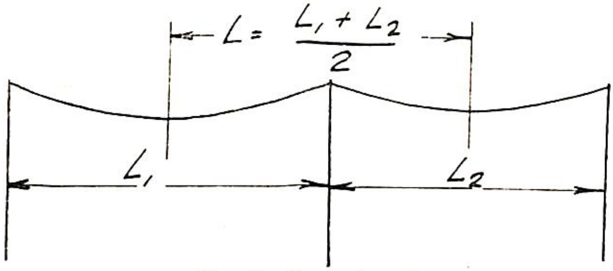

Fig. 12—Equivalent Span.

Example.

Design a support for the conductor in the previous example.

Minimum height of lower conductor

$$= \text{Ground clearance} + 122°\text{F. sag.}$$
$$= 20 + 5{\cdot}35$$
$$\text{Say} = 25\tfrac{1}{2} \text{ ft.}$$

For a 6 ft. symmetrical spacing the top conductor will be about $5\frac{1}{4}$ feet above the lower conductor, and allowing say 15 inches for a pin insulator the height of the pole above ground will be $29\frac{1}{2}$ feet.

As the pole size is not known, it is assumed that all load is due to the loaded conductors only, and a trial design worked out with an increased safety factor. The wind load has already been found and is 0·804 lbs. per foot run.

$$M_t = 0{\cdot}804 \times 30{\cdot}75 \times 350 = 8653 \text{ ft. lbs.}$$
$$M_e = 0{\cdot}804 \times 25{\cdot}5 \times 350 \times 2 = 14351 \text{ ft lbs.}$$
$$M = 23{,}004 \text{ ft. lbs.}$$

If a safety factor of 3·9 is allowed the permissible fibre stress is

$$F = \frac{7800}{3{\cdot}9} = 2000 \text{ lbs. per sq. in.}$$

$$\text{also } F = \frac{23{,}004 \times 12}{Z}$$

$$Z = \frac{23{,}004 \times 12}{2000} = 138$$

$$\text{Now } Z = \frac{\pi D^3}{32} \qquad D = \sqrt[3]{\frac{138 \times 32}{\pi}}$$
$$= 11{\cdot}2 \text{ ins.}$$

A pole $11\frac{1}{4}''$ diameter at ground level and say $6\frac{1}{2}''$ at the tip would be chosen. A check on the safety factor can now be made allowing for the wind load on the pole.

$$M_p = \frac{8 \times 29{\cdot}5 \times 29{\cdot}5 \times \{\ 11{\cdot}25 + (2 \times 6{\cdot}5)\ \}}{72}$$

$$= 2345 \text{ ft. lbs.}$$

$$M = 23{,}004 + 2345 = 25{,}349 \text{ ft. lbs.}$$

$$Z = \frac{\pi \times 11{\cdot}25^3}{32} = 139{\cdot}8$$

$$F = \frac{25{,}349 \times 12}{139{\cdot}8} = 2176 \text{ lbs. per sq. in.}$$

$$\text{Safety factor} = \frac{7800}{2176} = 3{\cdot}58$$

Thus a pole of this size first satisfies the safety factor requirements and will be suitable for normal spans, but for longer spans a stiffer pole will be required.

Crossarm Design.

Designers differ in opinion as to the relative merits of wooden and metal crossarms, but in practice there is little to choose between wood and steel. However, in recent years wooden crossarms have increased in popularity as full advantage can be taken of the insulating properties of the wood. A typical crossarm construction is shown in Fig. 11 (*a*). The arm is supported by a through bolt or a "U" bolt and in many cases angle braces are used.

For simple calculations the braces may be ignored.

$$\text{Bending moment at through bolt} = (W_i \times l_c) + (W_a \times L/4)$$

$$\text{Section modulus} = Z = \frac{b\ (d^3 - h^3)}{6d}$$

Where :—

W_i = Weight of ice covered conductor in lbs.
W_a = Weight of half the crossarm in lbs.
b, d, h, l_c are as shown in Fig. 11 (*a*) and (*b*).

Example.

Design a crossarm for the conductor and pole in the previous example. Weight of ice covered conductor is 0·896 lbs. per foot run.

As the weight of the crossarm is not yet known it must be assumed that all stress is produced by the conductor and insulator, and allowing a safety factor of, say, 4.

$$W_i = 350 \times 0{\cdot}896 = 314 \text{ lbs.} + \text{insulator weight.}$$
$$= 325 \text{ lbs. say.}$$

$$\text{Bending moment} = 325 \times 3{\cdot}6 = 11{,}700 \text{ in. lbs.}$$

$$\text{Permissible fibre stress} = \frac{7800}{4} = 1950$$

$$\text{Section modulus} = \frac{11{,}700}{1950} = 6$$

For a simple rectangular section $Z = \frac{b\,d^2}{6}$ and as the crossarm will have to be about $2\frac{1}{2}''$ wide to carry the insulator bolt the depth will be

$$d = \sqrt{\frac{Z \times 6}{b}} = \sqrt{\frac{6 \times 6}{2{\cdot}5}} = 3{\cdot}8 \text{ ins.}$$

A crossarm $2\frac{1}{2}'' \times 3\frac{3}{4}''$ should be suitable, and a check should now be made taking into account the weight of the crossarm and the through bolt hole.

Assuming the wood weighs 0·0174 lbs. per cub. in.

$$\text{Bending moment} = (325 \times 36) + (6{\cdot}9 \times 21).$$
$$= 11{,}700 + 145.$$
$$= 11{,}845 \text{ in. lbs.}$$

$$Z = \frac{2{\cdot}5\ (3{\cdot}75^3 - 1^3)}{6 \times 3{\cdot}75} = 5{\cdot}75$$ assuming a 1″ dia. hole for the through bolt.

$$\text{Fibre stress} = \frac{11{,}845}{5{\cdot}75} = 2060 \text{ lbs. per sq. in.}$$

$$\text{Safety factor} = \frac{7800}{2060} = 3{\cdot}78$$

This safety factor is well over the minimum of 3·5 and a crossarm of this size will be suitable.

Design of Crossarms for Terminal Poles.

A terminal pole crossarm must be able to withstand the stress due to the conductor tension as well as the weight of the ice covered conductor. These two loads act at 90° to each other in the vertical plane, and the simplest method is to design a crossarm that will withstand the stress due to conductor tension and then check that it will carry the ice loaded conductor.

A common method of gaining the extra strength is to use a double arm, a typical example being shown in Fig. 13.

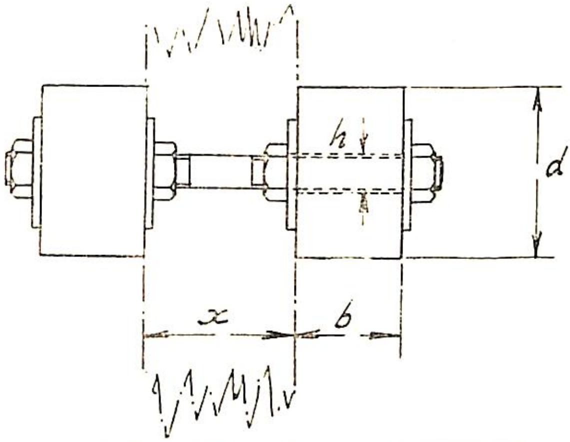

Fig. 13—Typical Double Crossarm Construction.

The strength of such an arm is about 30—40% of that of a solid arm of the same cross sectional area.

The section modulus for such an arrangement is

$$Z = \frac{(d-h)\ \{\ (2b+x)^3 - x^3\ \}}{6\ (2b+x)}$$

Example.

Design a terminal pole crossarm for the conductor in the previous example. The conductor tension is 3500 lbs.

$$\text{Permissible stress} = \frac{7800}{3{\cdot}5} = 2225 \text{ lbs. per sq. in.}$$

$$\text{Bending moment} = 3500 \times 36 = 126{,}000 \text{ in. lbs.}$$

Assuming a strength equal to 40% that of a solid arm, then

$$\text{Section modulus} = \frac{126{,}000}{{\cdot}4 \times 2225} = 141{\cdot}5$$

Assume that the thickness of the arm is 4″ spaced 6½″ apart and the hole diameter is 1″

$$d = 141{\cdot}5 \times \frac{6\,\{\ (2\times4) + 6{\cdot}5\ \}}{\{\ (2\times4) + 6{\cdot}5\}^3 - 6{\cdot}5^3} + 1$$

$$= 4{\cdot}45 + 1 = 5{\cdot}45''$$

Thus a crossarm made up as shown in Fig. 13 each part being 4″ × 5½″ will be suitable.

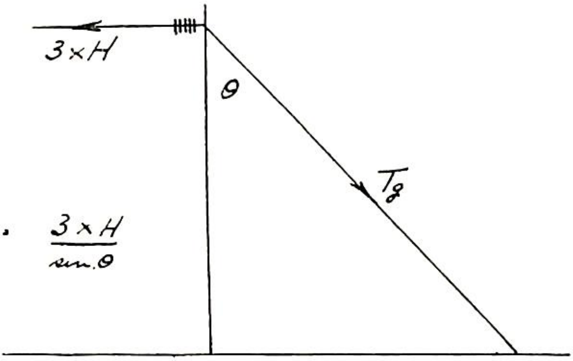

Fig. 14—Terminal Pole Guy.

Guys.

Guying is necessary at terminal poles and at angles in the line.

A typical guying arrangement for a terminal pole is shown in Fig. 14.

The horizontal tension = 3 × conductor tension for a three conductor line.

$$\text{Guy tension} = \frac{\text{Horizontal tension}}{\sin\ \theta}$$

An alternative arrangement is shown in Fig. 15 where overhead guying is used.

$$\text{Overhead guy tension} = \frac{\text{Horizontal tension}}{\sin\ \theta}$$

$$\text{Down guy tension} = \frac{\text{Horizontal tension}}{\sin\ \propto}$$

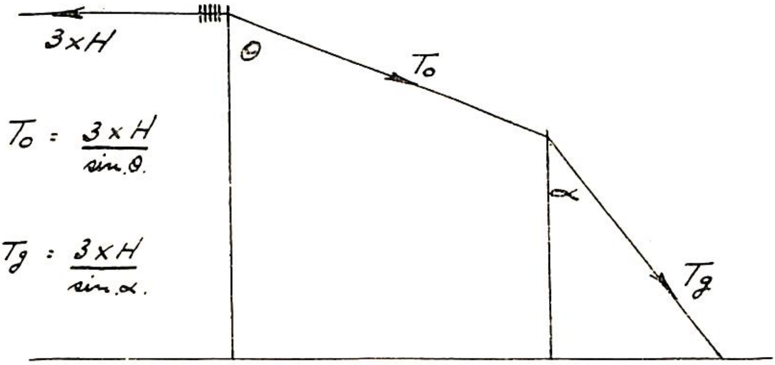

Fig. 15—Overhead Guy.

Fig. 16 shows an angle in the line and guys. The resultant load, $R = 6 \times H \times \sin \theta/2$ lbs.

Where H is the conductor tension and θ is the angle of deviation of the line.

$$\text{The guy tension} = \frac{R}{\sin \alpha} \text{ lbs.}$$

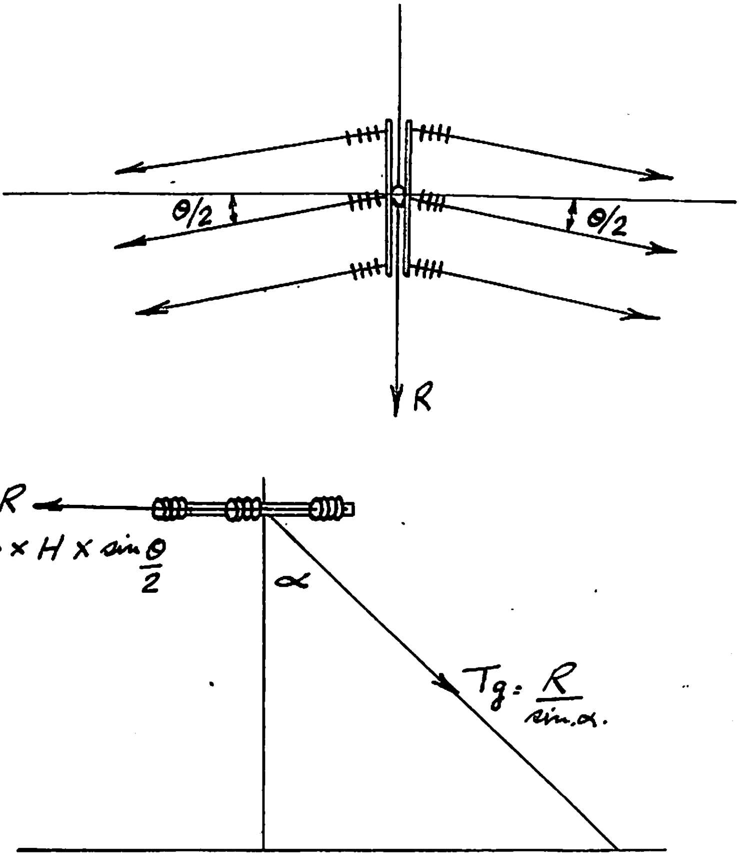
$\theta/2$
$\theta/2$
R
R
$= 6 \times H \times \sin \frac{\theta}{2}$
α
$T_g = \frac{R}{\sin.\alpha.}$

Fig. 16—Angle in Line.

"A" and "H" Poles.

There is some difference of opinion as to the relative strength of "A" and "H" poles, but they are certainly at least twice and possibly three times the strength of a single pole of the same size as one of the legs.

In practice the stresses, etc., are not calculated for the frames as such, but the safety factor for a single pole of the size to be used is found and multiplied by 2. For less important lines, the safety factor may be multiplied by 2½.

MISCELLANEOUS.

Insulator Design.

Insulator design is a specialist study in itself and cannot be entered into fully here, but one or two aspects of their design are of interest to the overhead line engineer.

For the type of line discussed in this pamphlet, pin type insulators are almost invariably used, a typical insulator being shown in Fig. 17.

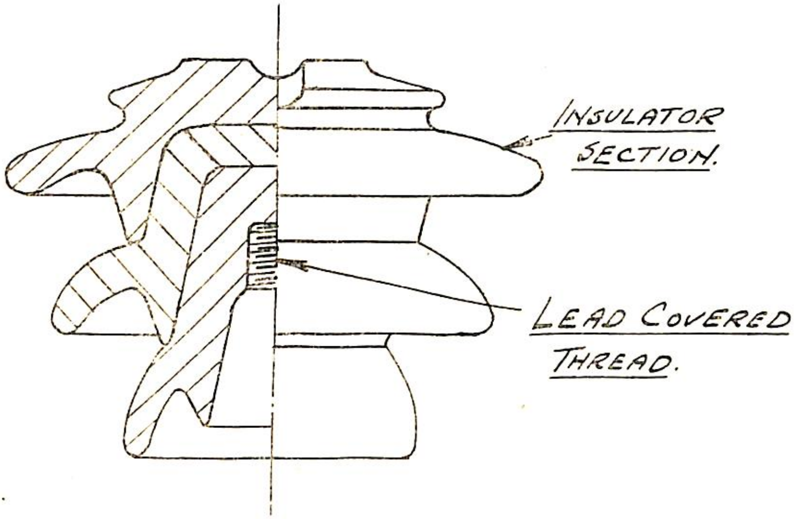

Fig. 17—Typical Pin Insulator.

Porcelain is the most common insulator material and has a weight of about 0·8 lbs. per cub. in. and a puncture strength of 320-700 kV. per inch, depending on the thickness of the sample, the lower value being for thinner material.

Insulators should be shaped so that the petticoats follow the equipotential lines and the main body the lines of electrostatic field as shown in Fig. 18.

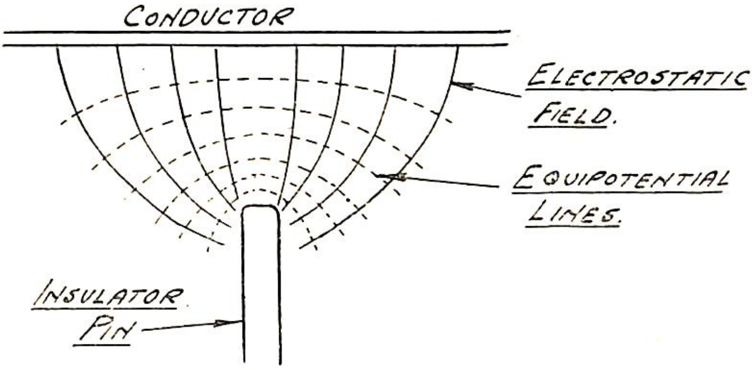

Fig. 18—Pin Insulator Fields.

The leakage distance is the shortest distance between the conductor and the pin over the surface of the insulator, and must be made long enough to prevent excessive leakage, Great care must be taken in this respect as large surfaces are kept clean by rain, but whilst it is actually raining it will become almost a conducting surface. Fog over towns and salt near the sea are also serious problems in reducing surface leakage resistance.

Conductor Vibration.

There are two main types of conductor vibration :—

1. Large amplitude vibration known as "dancing" and is thought to be due to ice forming on the conductors in an elliptical form and at an angle to the wind. The covered conductor will act as an aerofoil and experience a lift. Operation of the line is impossible as the conductors either touch each other or come within flashover distance of each other.
2. High frequency vibration with a very small amplitude is very common and occurs in light steady winds. The frequency of vibration is :—

$f = 3{\cdot}26 \times V/d$ cycles per sec.
where V = wind speed in m.p.h.
d = conductor diameter in inches.

The number of loops in the span due to this vibration is found from the following equation :—

$$l = \frac{1}{2f} \times \sqrt{\frac{H g}{w}}$$

where l = length of the loop in feet.
H = conductor tension in lbs.
w = conductor weight in lbs. per foot run.
g = 32·2.
f = frequency obtained in eqn. $f = 3{\cdot}26 \times V/d$.

This vibration can be damped out by means of various types of dampers, such as the Stockbridge, fixed to the conductor between two nodes where the amplitude of vibration is a maximum. They are usually fixed in the middle of the first loop.

Instead of dampers, reinforcing rods or a length of the same conductor are bound to the main conductor for a few feet on either side of the clamp, providing reinforcement at the point most liable to failure by fatigue.

SUMMARY OF DESIGN DATA.

Length of Line	20 miles
Design Load	2500 kW. at 0·8 p.f.
Line Voltage	33 kV. 3-phase
Current	54·7 amps.
Conductor Size	7/·152″ dia. Copper
Conductor Spacing	6 ft.
Conductor Resistance	0·1998 ohms. per 1000 yds.
Conductor Reactance	0·3488 ohms. per 1000 yds.
Sending End Voltage at Design Load	34·65 kV.
Regulation at Design Load	5%
Transmission Efficiency	94·4%
Ruling Span	350 ft.
Pole Length	35 ft. 5′6″ in ground
Pole Diameters	Ground level 11¼″. Tip 6½″
Crossarm Size	2½ × 3¾″
Conductor Weight	0·508 lbs. per foot run.
Ice Load on Conductor	0·388 lbs. per foot run
Wind Load on Conductor	0·804 lbs. per foot run
Sag under worst Load Conditions	5·268 ft. Tension 3500 lbs.
Sag at 122°F.	5·35 ft. Tension 1450 lbs.

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B.S.I. Specifications.

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B.S.S. 215—1934	Aluminium and Steel Cored Aluminium Conductor
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B.S.S. 1320—1946	High-voltage Overhead Lines on Wood Poles for Line Voltages up to and including 11 kV.

Electricity Commissioners' Publications.

El. C.	13	Public Safety and Efficient Supply.
El. C.	53	Design and Construction of O.H. Lines.

Books.

"Generation, Transmission and Utilization of Electrical Power."	A. T. Starr.	Pitman.
"Sags and Tension in O.H. Lines."	C. G. Watson.	Pitman.

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21. Particulars for Balata Belt Drives.
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23. 1" ,, ,, ,, ,,
24. ¾" Sq. Steel Tubes as Struts (30 ton yield).
25. ⅞" ,, ,, ,, (30 ton yield).
26. 1" ,, ,, ,, (30 ton yield).
27. ¾" ,, ,, ,, (40 ton yield).
28. ⅞" ,, ,, ,, (40 ton yield).
29. 1" ,, ,, ,, (40 ton yield).
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31. Moments of Inertia of Built-up Sections (Instructions and Examples). } Connected.
32. Reinforced Concrete Slabs (Line Chart).
33. Reinforced Concrete Slabs (Instructions and Examples) } Connected.
34. Capacity and Speed Chart for Troughed Band Conveyors.
35. Screw Propeller Design (Sheet 1, Diameter Chart).
36. ,, ,, ,, (Sheet 2, Pitch Chart).
37. ,, ,, ,, (Sheet 3, Notes and Examples) } Connected.
38. Open Coil Conical Springs.
39. Close Coil Conical Springs.
40. Trajectory Described by Belt Conveyors (Revised 1949),
41. Metric Equivalents.
42. Useful Conversion Factors.
43. Torsion of Non-Circular Shafts.
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52. Approximate Radius of Gyration of Various Sections.

53. Helical Spring Graphs (Round Wire).
54. ,, ,, ,, (Round Wire). Connected.
55. ,, ,, ,, (Square Wire).
56. Relative Value of Welds to Rivets.
58. Graphs for Strength of Rectangular Flat Plates of Uniform Thickness.
59. Graphs for Deflection of Rectangular Flat Plates of Uniform Thickness.
60. Moment of Resistance of Reinforced Concrete Beams.
61. Deflection of Leaf Spring.
62. Strength of Leaf Spring.
63. Chart Showing Relationship of Various Hardness Tests.
64. Shaft Horse Power and Proportions of Worm Gear.
65. Ring with Uniform Internal Load (Tangential Strain) Connected.
66. Ring with Uniform Internal Load (Tangential Stress)
67. Hub Pressed on to Steel Shaft. (Maximum Tangential Stress at Bore of Hub).
68. Hub Pressed on to Steel Shaft. (Radial Gripping Pressure between Hub and Shaft).
69. Rotating Disc (Steel) Tangential Strain. Connected.
70. ,, ,, ,, Stress.
71. Ring with Uniform External Load, Tangential Strain. Connected.
72. ,, ,, ,, ,, ,, Stress.
73. Viscosity Temperature Chart for Converting Commercial to Absolute Viscosities. Connected.
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93-4-5-6-7. Rolled Steel Sections.
98-99-100. Boiler Safety Valves.
102. Pressure Required for Blanking and Piercing.
103. Punch and Die Clearances for Blanking and Piercing.
104. Nomograph for Valley Angles of Hoppers and Chutes.
105. Permissible Working Stresses in Mild Steel Struts to B.S. 449, 1948.

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